SPECIAL IMPORTANT QUESTIONS

JR MATHS-IB

LOCUS

Short Answer type questions

***1. A (5, 3) and B (3, -2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq. units.

***2. Find the equation of the locus of a point P such that the distance of P from the origin is twice the distance of P from A(1,2).

***3. If the distance from P to the points (2,3), (2,-3) are in the ratio 2:3 then find the equation of locus of P.

***4. Find the equation of the locus of a point, which forms a triangle of area 2, with the points A(1,1) and $B(-2,3)$. **(IN TEXTBOOK)**

***5. Find the equation of locus of P, if the ratio of the distances from P to the points $A(5,-4)$ and $B(7,6)$ is 2:3.

6. Find the equation of the locus of a point whose from $A(4,-3)$ is 5 units. **(IN TEXTBOOK)**

TRANSFORMATION OF AXES

***1. When the axes are rotated through an angle α, find the transformed equation of $x\cos\alpha + y\sin\alpha = p$

***2. When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

***3. When the axes are rotated through an angle $\frac{\pi}{4}$ find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.

***4. When the origin is shifted to (-1,2) by the translation of axes, find the transformed equation of

i) $2x^2 + y^2 - 4x + 4y = 0$ ii) $x^2 + y^2 + 2x - 4y + 1 = 0$

***5. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$

6. When the origin is shifted to the point (2,3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.

STRAIGHT LINES (Q.NO-1 & 2)

Very Short Answer type questions

***1. Find the ration in which the straight line $2x + 3y - 20 = 0$ divides the join of the points (2,3) and (2,10)

***2. Find the value of y, if the line joining (3,y) and (2,7) is parallel to the line joining the points (-1,4) and (0,6)

***3. Find the equation of line passing through the points $(2,-3)$ and $(0,-3)$. **(IN TEXTBOOK)**

***4. Find the sum of the squares of the intercepts of the line $4x - 3y = 12$ on the axes of coordinates

5. Find the condition for the points $(a,0),(h,k)$ and $(0,b)$ where $ab \neq 0$ to be collinear

6. Find the equation of line containing the points $(1,2)$ and $(1,-2)$. **(IN TEXTBOOK)**

7. Find the slope of the straight line passing through the points $(-3,8)$ and $(10,5)$ **(IN TEXTBOOK)**

8. Transform $\sqrt{3}x + y + 10 = 0$ into normal form **(IN TEXTBOOK)**

9. Find the ratio in which the given straight line divides the line segment of joining the given points. Also state whether the points lie on the same side or on either side of the straight line
$3x-4y=7$; $(2,-7)$ *and* $(-1,3)$ **(IN TEXTBOOK)**
10. Find the point of intersection the lines **(IN TEXTBOOK)**
i) $4x+8y-1=0,\ 2x-y+1=0$ ii) $7x+y+3=0,\ x+y=0$
11. Find the length of the perpendicular drawn from the point (-2, -3) to the straight line 5x–2y+4=0
12. Find the perpendicular distance from the point (3,4) to the straight line $3x-4y+10=0$
13. Find the distance between the parallel straight lines
i) $3x+4y-3=0$ and $6x+8y-1=0$ ii) $3x-4y=12$ and $3x-4y=7$
14. Find the equation of the straight line passing through (-4, 5) and making non-zero intercepts on the co-ordinate axes whose sum is zero.
15. Transform into Normal form $x+y+1=0$
16. If the product of the intercepts made by line $x\tan\alpha+y\sec\alpha=1$ is $\sin\alpha$, then find α.
17. Find the area made by the straight line $3x-4y+12=0$ on coordinate axes.

Short Answer type questions

***1. Prove that the points (1,11), (2,15), (-3,-5) are collinear and find the equation of the straight line containing them.
***2. Find the value of x if the slope of the line passing through (2,5) and (x,3) is 2.
***3. Find the equation of the straight line passing through (2, 3) and making non-zero intercepts on the co-ordinate axes whose sum is zero.
4. Find the value of P, if the straight lines x + p = 0, y + 2 = 0, 3x + 2y + 5 = 0 are concurrent.
5. Find the value of p, if the lines $3x+4y=5, 2x+3y=4$ and $px+4y=6$ are concurrent
6. Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form when a>0 and b>0.If the perpendicular distance of straight line from the origin is p, deduce that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$
7. Transform the equation $3x+4y+12=0$ into (a) slope-intercept form (b) intercept form and (c) normal form.
8. Find the equation of the straight lines passing through the origin and also passing through the point of intersection of $2x-y+5=0$ and $x+y+1=0$. **(IN TEXTBOOK)**
9. Find the equation of the straight line parallel to the line 3x + 4y = 7 and passing through the point of intersection of the lines x - 2y – 3 = 0 and x + 3y - 6 = 0.
10. Find the equation of the straight line perpendicular to the line 2x + 3y = 0 and passing through the point of intersection of the lines x + 3y – 1 = 0 and x – 2y + 4 = 0.

Long Answer type questions

***1. Find the orthocentre of a triangle, whose vertices are (-2,-1), (6,-1) and (2,5)
***2. If p and q are length of the perpendicular from origin to the straight lines $x\sec\alpha+y\operatorname{cosec}\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos 2\alpha$ then prove that $4p^2+q^2=a^2$
***3. If Q(h,k) is the foot of the perpendicular from p(x_1,y_1) on the straight lines ax+by+c=0 then prove that $(h-x_1):a=(k-y_1):b=-(ax_1+by_1+c):a^2+b^2$, find the foot of the perpendicular from $(-1,3)$ on the straight line $5x-y-18=0$.
4. Find the circumcentre of the triangle whose vertices are (1,0), (-1,2) and (3,2)

PAIR OF STRAIGHT LINES

Long Answer type questions

***1. Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is $\frac{n^2\sqrt{h^2-ab}}{|am^2-2hlm+bl^2|}$ sq. units.

***2. Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\frac{|c|}{\sqrt{(a-b)^2+4h^2}}$

***3. If the equation $S=ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines, then show that $(i)\,h^2=ab$ $(ii)\,af^2=bg^2$ and (iii) the distance between the parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}}=2\sqrt{\frac{f^2-bc}{b(a+b)}}$

***4. Find the values of k, if the lines joining the origin to the points of intersection of the curve $2x^2-2xy+3y^2+2x-y-1=0$ and the line x + 2y = k are mutually perpendicular.

***5. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2+2xy+y^2+2x+2y-5=0$ and the line 3x – y + 1 = 0.

***6. Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ (whose centre is the origin) to subtend a right angle at the origin.

7. Let the equation $ax^2+2hxy+by^2=0$ represents a pair of straight lines. Then the angle θ between the lines is given by $\cos\theta=\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$

8. Show that the product of the perpendicular distances from a point (α,β) to the pair of straight lines $ax^2+2hxy+by^2=0$ is $\frac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$

9. Find the angle between the straight lines joining the origin to the points of intersection of the curve $\mathbf{7x^2-4xy+8y^2+2x-4y-8=0}$ with the straight line $\mathbf{3x-y=2}$

3D – GEOMETRY

Very Short Answer type questions

1. Find the *x*, if the distance between (5, -1,7) and (*x*,5,1) is 9.
2. Find the centroid of the triangle whose vertices are (5,4,6),(1,-1,3) and (4,3,2).
3. Find the ratio in which XZ plane divides the lines joining $A(-2,3,4)$ and $B(1,2,3)$
4. Find the distance between the points $(3,4,-2)$ *and* $(1,0,7)$
5. Show that the points A(1, 2, 3), B(7, 0, 1), C (–2, 3, 4) are collinear
6. Find the distance of $P(3,-2,4)$ from the origin
7. Find the centroid of the tetrahedron whose vertices are $(2,3,-4),(-3,3,-2),(-1,4,2),(3,5,1)$

D.C'S & D.R'S

Long Answer type questions

***1. Find the angle between the lines whose direction cosines are given by the equations 3*l* + m + 5n = 0 and 6mn – 2n*l* + 5*l*m = 0

***2. Find the angle between two diagonals of a cube.

***3. The vertices of triangle are A(1,4,2), B(-2,1,2). C(2,3-4)). Find $\angle A, \angle B, \angle C$

***4. Find the angle between the lines whose direction cosines satisfy the equations $l + m + n = 0,\ l^2 + m^2 - n^2 = 0$

5. Find the direction cosines of two lines which are connected by the relations *l+m+n=0 and mn-2nl-2lm=0*

6. Find the direction cosines of two lines which are connected by the relations *l-5m+3n=0 and* $7l^2 + 5m^2 - 3n^2 = 0$

PLANES

Very Short Answer type questions

1. Transform the equation 4x - 4y + 2z + 5=0 into intercept form
2. Find the equation of the plane whose intercepts on X, Y, Z - axes are 1, 2,4 respectively.
3. Find the intercepts of the plane 4x + 3y – 2z + 2 = 0 on the coordinate axes.
4. Reduce the equation x + 2y – 3z – 6 = 0 of the plane in to the normal form
5. Find the angle between the planes x + 2y + 2z – 5 = 0 and 3x + 3y + 2z – 8 = 0.
6. Find the equation of the plane passing through the point (1,1,1) and parallel to the plane x + 2y + 3z – 7 =0.

LIMITS

Very Short Answer type and Short Answer type questions

1. Evaluate $\lim_{x\to 2}\left(\frac{x-2}{x^3-8}\right)$
2. Evaluate $\lim_{x\to 0}\frac{e^{7x}-1}{x}$
3. Evaluate $\lim_{x\to 3}\frac{e^x-e^3}{x-3}$
4. Evaluate $\lim_{x\to\infty}\frac{3x^2+4x+5}{2x^3+3x-7}$ **(IN TEXTBOOK)**
5. Evaluate $Lt_{x\to 0}\frac{e^{3x}-1}{x}$
6. Compute $Lt_{x\to\infty}\frac{2x^2-x+3}{x^2-2x+5}$
7. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x}$
8. Evaluate $\lim_{x\to a}\frac{x^2-a^2}{x-a}$ **(IN TEXTBOOK)**
9. Evaluate $\lim_{x\to 1}\left(\frac{2x+1}{3x^2-4x+5}\right)$
10. Evaluate $\lim_{x\to 2}\left(\frac{2}{x+1}-\frac{3}{x}\right)$
11. Evaluate $\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}+1}\right)$
12. Evaluate $\lim_{x\to 2^-}\frac{|x-2|}{x-2}=-1$

13. Evaluate $\lim_{x \to \infty} \frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}$

14. Evaluate $\underset{x \to \infty}{Lt} \frac{x^2 + 5x + 2}{2x^2 - 5x + 1}$

15. Evaluate $\lim_{x \to \infty} \frac{8|x| + 3x}{3|x| - 2x}$

16. Is the function *f*, defined by $f(x) = \begin{cases} x^2 \ if \ x \le 1 \\ x \ if \ x > 1 \end{cases}$ continuous on R?

17. Is the function *f*, defined by $f(x) = \begin{cases} \frac{\sin 2x}{x}, if \ x \neq 0 \\ 1 \quad\quad , if \ x = 0 \end{cases}$ continuous at x=0

18. Evaluate $\lim_{x \to 0} \left(\frac{\sqrt{1+x} - 1}{x} \right)$

19. Evaluate $\underset{x \to 1}{Lt} \frac{\sin(x-1)}{x^2 - 1}$

20. $\underset{x \to a}{L\lim} \left[\frac{x \sin a - a \sin x}{x - a} \right]$

21. $\underset{x \to 0}{L\lim} \left[\frac{(1+x)^{\frac{1}{8}} - (1-x)^{\frac{1}{8}}}{x} \right]$

22. Evaluate $\lim_{x \to 2} \frac{(2x^2 - 7x - 4)}{(2x-1)(\sqrt{x} - 2)}$

23. Evaluate $\lim_{x \to 3} \frac{x^3 - 6x^2 + 94}{x^2 - 9}$

24. Evaluate $\lim_{x \to 3} \frac{x^2 - 8x + 15}{x^2 - 9}$

CONTINUITY

***1. Evaluate $\underset{x \to 1}{Lim} \frac{(2x-1)(\sqrt{x} - 1)}{2x^2 + x - 3}$

***2. Evaluate $\underset{x \to 0}{lim} \frac{1 - \cos mx}{1 - \cos nx}, n \neq 0$

***3. Evaluate $\lim_{x \to 0} \left(\frac{\cos ax - \cos bx}{x^2} \right)$

DIFFERENTIATION

Very Short Answer type questions

Find the derivatives of the following

1. $5 \sin x + e^x \log x$
2. $\sin^{-1}(3x - 4x^3)$

3. $\log\left(\sin^{-1}\left(e^{x}\right)\right)$

4. $\dfrac{1-\cos 2x}{1+\cos 2x}$

5. $\cot^{nx}$

6. $5^{x}+\log x+x^{3}e^{x}$

7. $e^{x}+\sin x\cos x$

8. $\sin^{-1}\left(\cos x\right)$

9. $x\tan^{-1}x$

10. $\sqrt{2x-3}+\sqrt{7-3x}$

11. $y=\sin\left(\log x\right)$, then find $\dfrac{dy}{dx}$

12. $\tan^{-1}\left(\log x\right)$

13. $\cos(\log x+e^{x})$ then find $\dfrac{dy}{dx}$

14. If $y=\sin^{-1}\left(\sqrt{x}\right)$ then find $\dfrac{dy}{dx}$

15. If $\tan\left(e^{x}\right)$ then find $\dfrac{dy}{dx}$

16. If $f(x)=1+x+x^{2}+.....+x^{100}$ then find $f^{1}(1)$

17. If $\left(x^{3}+6x^{2}+12x-13\right)^{100}$ then find $\dfrac{dy}{dx}$

18. If $y=\log\left(\sin\left(\log x\right)\right)$ then find $\dfrac{dy}{dx}$

Short Answer type questions

Find the derivatives of the following from first principle of derivatives

1. $\sqrt{x+1}$
2. $\sin 2x$
3. $\tan 2x$
4. $\cot x$
5. $\sec 3x$
6. $ax^{2}+bx+c$
7. $\cos^{2}x$
8. If $x=a\left[\cos t+\log\tan\left(\dfrac{t}{2}\right)\right], y=a\sin t$ then find $\dfrac{dy}{dx}$

Long Answer type questions

***1. If $x^{y}+y^{x}=a^{b}$ then show that $\dfrac{dy}{dx}=-\left[\dfrac{yx^{y-1}+y^{x}\log y}{x^{y}\log x+xy^{x-1}}\right]$

***2. If $y=Tan^{-1}\left[\dfrac{\sqrt{1+x^{2}}+\sqrt{1-x^{2}}}{\sqrt{1+x^{2}}-\sqrt{1-x^{2}}}\right]$ for $0<|x|<1$, find $\dfrac{dy}{dx}$

***3. Find the derivative of $\left(\sin x\right)^{x}+x^{\sin x}$ with respective x.

***4. If $y=x^{\sin x}+\left(\sin x\right)^{x}$, find $\dfrac{dy}{dx}$.

5. If $x^y = y^x$ then show that $\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$

6. Find the derivative of $y = x^{\tan x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$

APPLICATION OF DERIVATIVES

Very Short Answer type questions

1. Find Δy and dy for the function, if $y = 5x^2 + 6x + 6$, where x = 2 and $\Delta x = 0.001$
2. Find Δy and dy for $y = f(x) = x^2 + x$ at x = 10 and $\Delta x = 0.1$
3. Find Δy and dy for $y = x^2 + 3x + 6$ at x = 10 and $\Delta x = 0.01$
4. Find the approximations of the following i) $\sqrt{25.001}$ ii) $\sqrt[3]{7.8}$.
5. Find the approximate value of $\sqrt{82}$
7. Find the approximate value of $\sqrt[3]{65}$
8. The side of a square is increasing 3 cm to 3.01 cm. Find the approximate increased in its area. **(IN TEXTBOOK)**
10. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$
11. Show that at any point (x, y) on the curve $y = be^{x/a}$, the length of the sub-tangent is a constant and the length of the sub normal is $\frac{y^2}{a}$
12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?
13. Find the value of 'c' from Rolle's theorem for the function $f(x) = x^2 - 1$ on $[-1,1]$
14. Find the value of 'c' from Rolle's theorem for the function $y = f(x) = x^2 + 4$ on $[-3,3]$
15. Find 'c' so that $f'(c) = \frac{f(b) - f(c)}{b - a}$ in the $f(x) = e^x; a = 0, b = 1$
16. Find the points of local extrema (if any) and local extrema for $f(x) = x^2 \; \forall x \in R$
17. Find the slope of tangent of the curve $y = 5x^2$ at $(-1,5)$

Short Answer type questions

1. The diameter of the sphere is measured to be 40cm. if an error of 0.02cm is made in it, then find the approximate errors in volume and surface area of the sphere.
2. The radius of a sphere is measured as 14cm later it was found that there is an error 0.2cm in measuring the radius. Find the approximate error in surface area of the sphere. **(IN TEXTBOOK)**
3. The total revenue in rupees received from the sale of *x* units of a produce is given by $R(x) = 13x^2 + 26x + 15$, Find the marginal revenue when x=7.
4. The distance time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$, find when and where the velocity is zero.
5. At time *t*, the distance *s* of a particle moving in a straight line is given by $S = -4t^2 + 2t$. Find the average velocity between t=2 sec and t=8 sec.
6. Show that at any point (x, y) on the curve $y = be^{x/a}$, the length of sub-tangent is constant and the length of the sub normal is $\frac{y^2}{a}$.

7. Find the length of sub tangent and subnormal at a point on the curve $y = b\sin\frac{x}{a}$.

8. Find the lengths of normal and sub normal at a point on the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$

9. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ $(a \neq 0, b \neq 0)$ at the point (a,b) is

$\frac{x}{a} + \frac{y}{b} = 2$

10. Find the equations of tangent and normal to the curve $y = x^3 + 4x^2$ at (-1, 3)

11. A particle is moving along a straight line has the relation $S = t^3 + 2t + 3$, connecting the distance S and time *t*. Find the velocity and accelerates of the particle at $t = 4$ sec.

12. Find the equations of the tangent and normal to the curve $y = 5x^4$ at the point (1, 5).

13. Find the slope of the tangent of the curve $y = x^3 - x + 1$ whose x – coordinate is 2.

14. Find the sloe pf the tangent of the curve $y = x^3 - 3x + 2$ whose x – coordinate is 3.

15. Find the equation of tangent and normal of the curve is $y = x^3 + 4x$ at $(-1,3)$

16. A particle is moving $S = t^3 + 9t^2 + 2yt - 18$. Find when and where velocity is zero.

Long Answer type questions

***1. Find the angle between the curves $y^2 = 4x$ and $x^2 + y^2 = 5$

***2. Find the angle between the curves $x + y + 2 = 0, x^2 + y^2 - 10y = 0$

***3. Find the angle between the curves $y^2 = 8x$ and $4x^2 + y^2 = 32$

***4. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$

***5. From a rectangular sheet of dimensions 30cm 80cm. four equal squares of side x cm. are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.

***6. If the curved surface of right circular cylinder inscribed in a sphere of radius *r* is maximum show that the height of the cylinder is $\sqrt{2}\,r$.

***7. A wire of length *l* is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the area is the least.

8. Find the angle between the curve xy=2 and $x^2 + 4y = 0$

9. If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the coordinate axes in A and B, then show that the length AB is a constant.

10. Show that the equation of tangent at the point (x_1, y_1) on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $xx_1^{-1/2} + yy_1^{-1/2} = a^{1/2}$

11. Find the length of sub tangent, subnormal at a t on the curve $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$

12. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20ft., find the maximum area.

13. The profit function $f(x)$ of a company selling x items per day is given by $p(x) = x(150 - x) - 1600$ Find the number of items that the company should sell to get maximum profit. Also find the maximum profit.

14. A manufacturer can sell *x* items at a price of rupees (5 - *x*/100) each. The cost price of *x* items is Rs. (*x*/5 + 500). Find the number of items that the manufacturer should sell to earn maximum profits.